A Stochastic Electricity Market Setting with Fair Pricing Properties

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Day-Ahead & Real-Time Markets

Sequence of Events

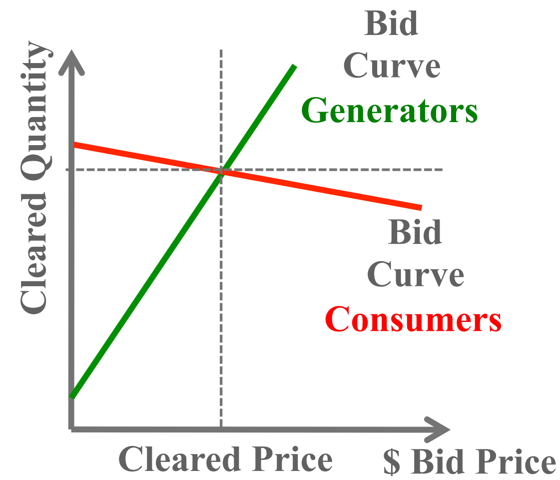
1.Players Provide Bid Prices to ISO using Forecasts of Prices & Capacities for Next Day

2.ISO Solves Day-Ahead Clearing using Bids and Expected Topology to Set Day-Ahead Quantities & Prices

3.Players Re-Bid using Observed Prices & Capacities for Next Hour

4.ISO Solves Real-Time Clearing to Set Real-Time Quantities & Prices

5.Players Correct Schedules & Get Paid for Day-Ahead Quantities & Real-Time Corrections



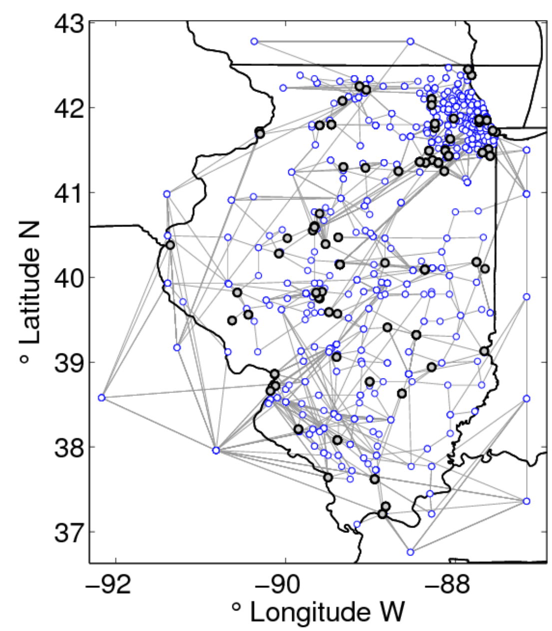
ISO (Independent System Operator) Metrics

▪**Social Surplus**
- Efficient Allocation of Physical Assets (Plants, Network)

▪**Price Predictability**
- Day-Ahead & Real-Time Prices Converge (Or Remain Close)

▪**Revenue Adequacy**
- ISO Does Not Run Into Financial Deficit

▪**Fairness**
- ISO Does Not Interfere (Biases) Market Transactions



Deterministic Day-Ahead Clearing

Bid Prices

Day-Ahead Prices

$$\min_{d_j, g_i, f_\ell} \sum_{i \in \mathcal{G}} \alpha_i^g g_i - \sum_{j \in \mathcal{D}} \alpha_i^d d_j$$

Social Surplus

$$\text{s.t.} \sum_{\ell \in \mathcal{L}_n^{rec}} f_\ell - \sum_{\ell \in \mathcal{L}_n^{snd}} f_\ell + \sum_{i \in \mathcal{G}_n} g_i - \sum_{i \in \mathcal{D}_n} d_i = 0, \quad (\pi_n) \quad n \in \mathcal{N}$$

Network Balance

$$-\bar{f}_\ell \le f_\ell \le \bar{f}_\ell, \qquad \ell \in \mathcal{L}$$

Transmission Limits

$$0 \le g_i \le \bar{g}_i, \qquad i \in \mathcal{G}$$

Generation Limits

$$0 \le d_j \le \bar{d}_j, \qquad j \in \mathcal{D}$$

Demand Limits

Expected Capacity Constraints

Deterministic Day-Ahead Clearing

Bid Prices

Day-Ahead Prices

$$\min_{d_j,g_i,f_\ell} \quad \sum_{i\in\mathcal{G}} \alpha_i^g g_i - \sum_{j\in\mathcal{D}} \alpha_i^d d_j$$

Social Surplus

$$\text{s.t.} \sum_{\ell\in\mathcal{L}_n^{rec}} f_\ell - \sum_{\ell\in\mathcal{L}_n^{snd}} f_\ell + \sum_{i\in\mathcal{G}_n} g_i - \sum_{i\in\mathcal{D}_n} d_i = 0, \quad (\pi_n) \quad n\in\mathcal{N}$$

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Generation Limits

$$0 \le d_j \le \bar{d}_j, \qquad j\in\mathcal{D}$$

Demand Limits

Expected Capacity Constraints

Deterministic Real-Time Clearing

$$\min_{D_j(\cdot),G_i(\cdot),F_\ell(\cdot)} \quad \sum_{i\in\mathcal{G}} \left(\alpha_i^{g,+}(G_i(\omega)-g_i)_+ - \alpha_i^{g,-}(G_i(\omega)-g_i)_-\right)$$

$$-\sum_{j\in\mathcal{D}} \left(\alpha_j^{d,+}(D_j(\omega)-d_j)_+ - \alpha_j^{d,-}(D_j(\omega)-d_j)_-\right)$$

Bid Prices

Real-Time Prices

$$\text{s.t.} \sum_{\ell\in\mathcal{L}_n^{rec}} F_\ell(\omega) - \sum_{\ell\in\mathcal{L}_n^{snd}} F_\ell(\omega) + \sum_{i\in\mathcal{G}_n} G_i(\omega) - \sum_{j\in\mathcal{D}_n} D_j(\omega) = 0, \ (\Pi_n(\omega)), \ n\in\mathcal{N}$$

$$-\bar{F}_\ell(\omega) \le F_\ell(\omega) \le \bar{F}_\ell(\omega), \qquad \ell\in\mathcal{L}$$

$$0 \le G_i(\omega) \le \bar{G}_i(\omega), \qquad i\in\mathcal{G}$$

$$0 \le D_j(\omega) \le \bar{D}_j(\omega), \qquad j\in\mathcal{D}$$

Real-Time Capacity Constraints

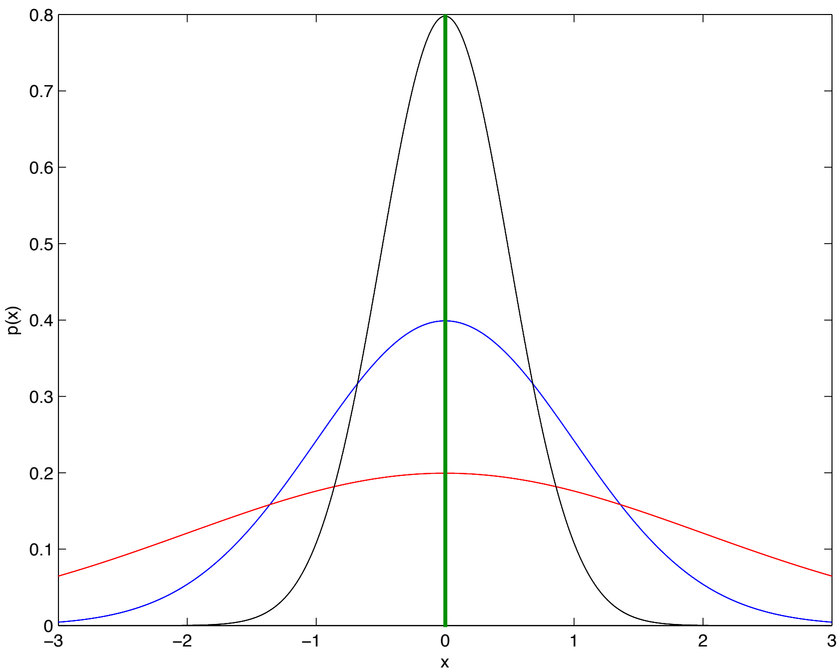
Possible Scenarios

$\forall\, \omega\in\Omega$

Limitations of Deterministic Market Setting

- Players are Forced to <u>Summarize</u> Uncertain Information using a Single Statistic
(e.g., Expected Value or Worst-Case Forecast)

$$\bar{g}_{i,t} = \mathbb{E}[\bar{G}_{i,t}(\omega)]$$
$$\bar{d}_{i,t} = \mathbb{E}[\bar{D}_{i,t}(\omega)]$$



- Day-Ahead Prices Do Not “Factor In” Uncertainty

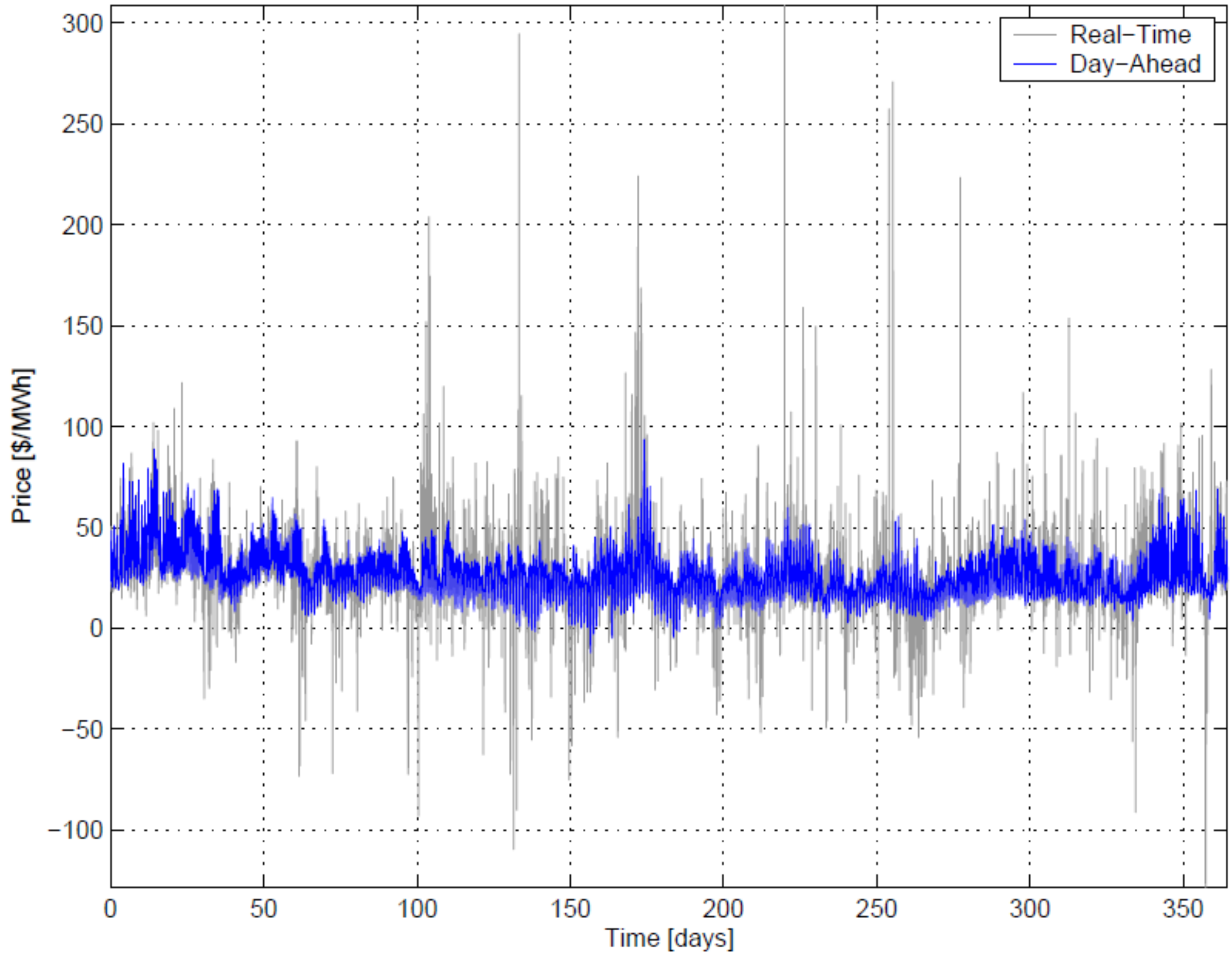
- Real-Time Prices Do Not Converge to Day-Ahead Prices

- Prices and Payments are Biased :
Only a Subset of Players are Benefited

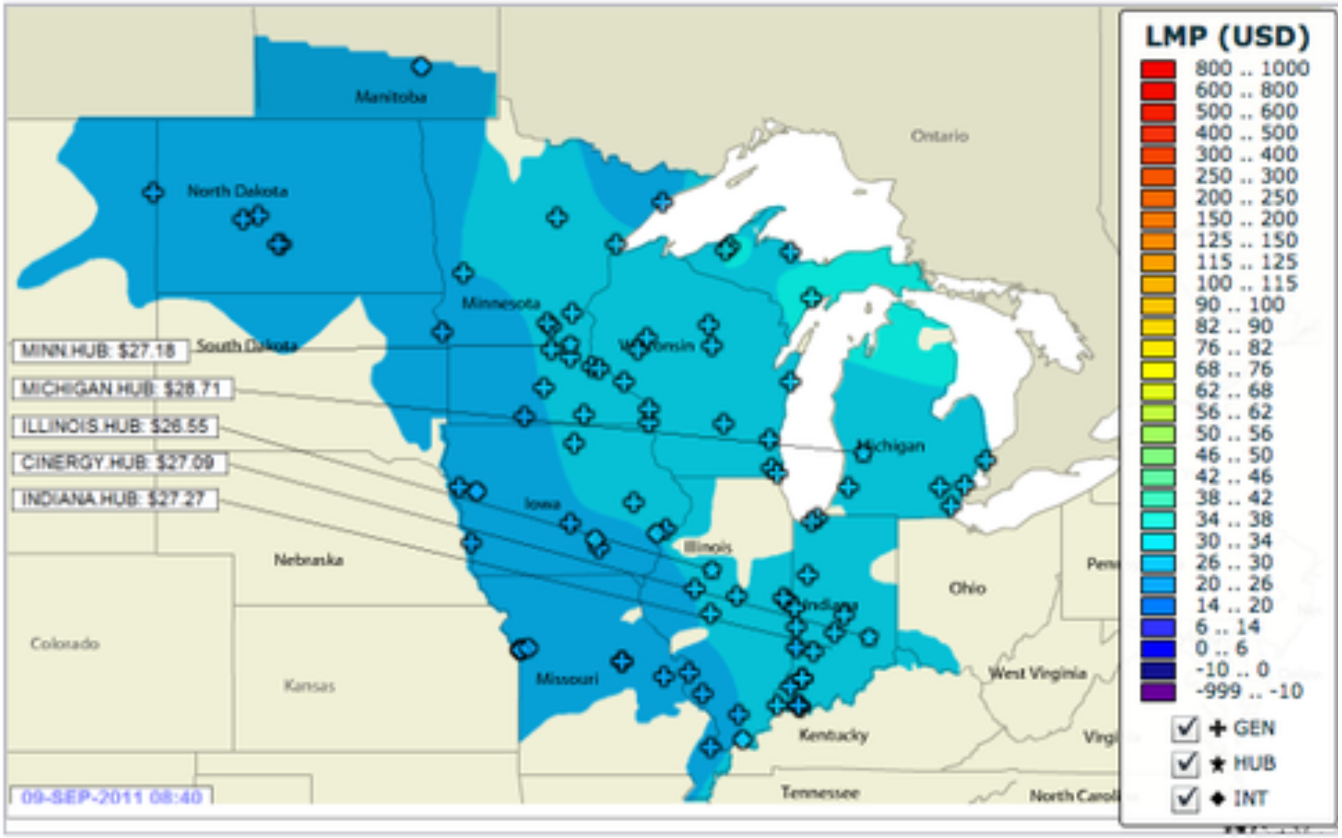
- Blocks Entry of New Participants (Affects Diversification) :
e.g., Wind, Price-Responsive Demands as we will show in the simulations.

Temporal Price Volatility

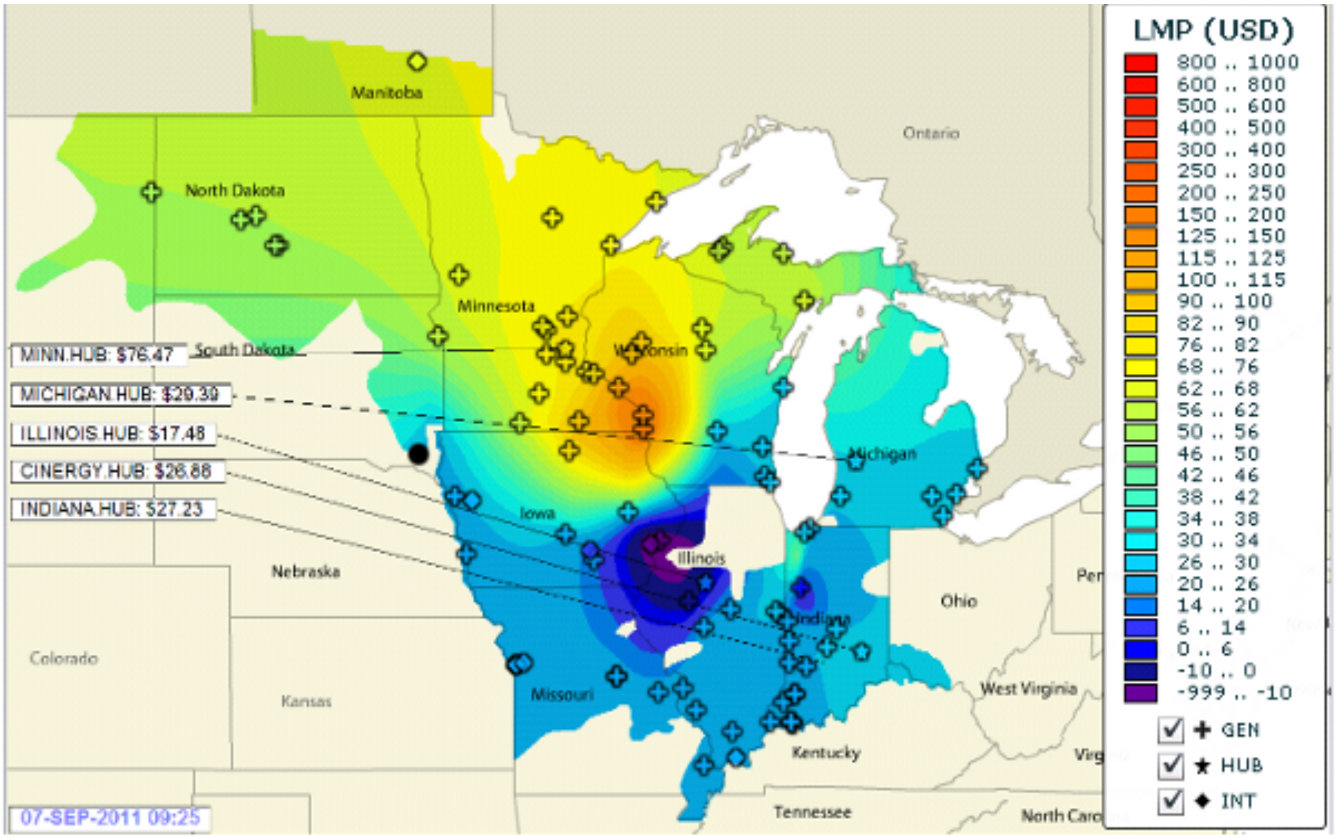
Prices at Illinois Hub, 2009



Spatial Price Volatility

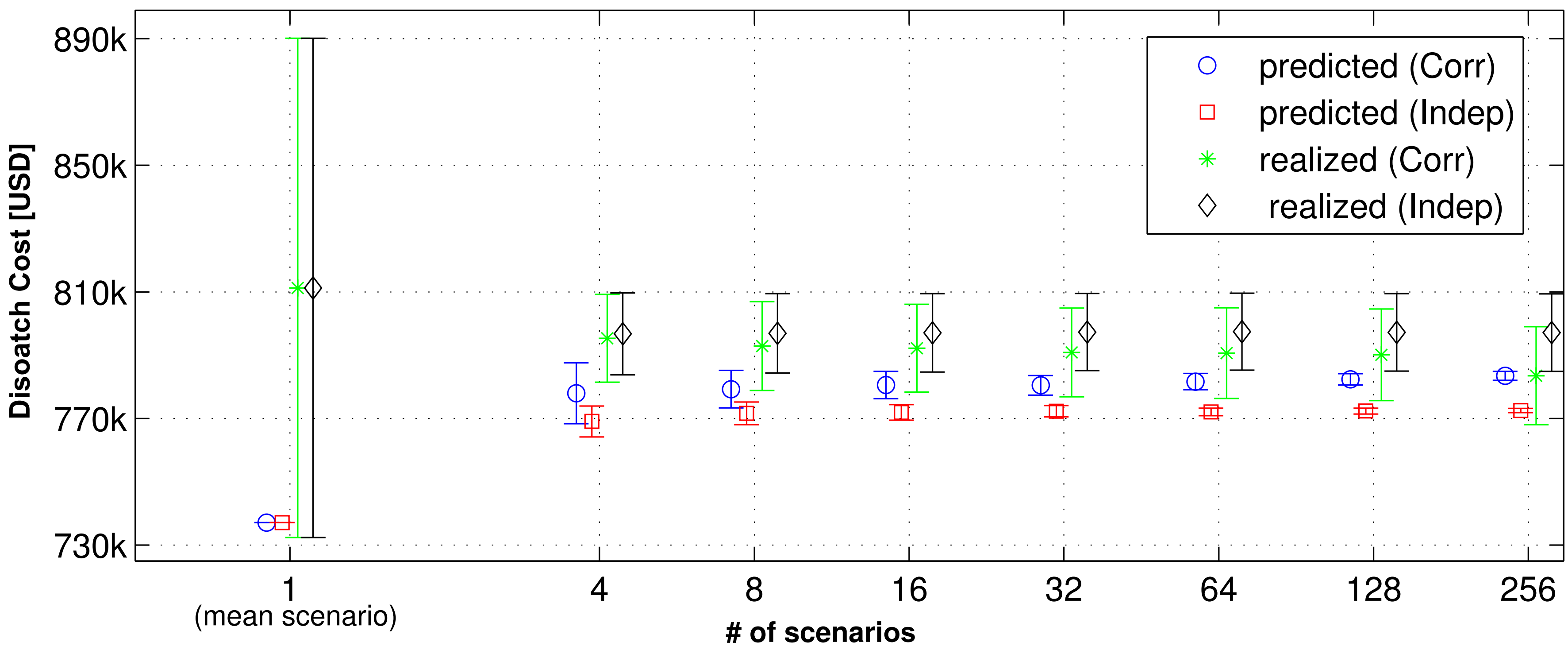


Spatial Price Volatility



Lost Opportunity when Ignoring Stochastic Structure

In one study with 17% wind penetration and full network over Illinois, we see 5% opportunity loss for deterministic dispatch and 1-2% for incorrect correlation modeling (see Petra talk)



The Message(s)

I) Deterministic clearing introduces strong distortions between day-ahead & expected real-time prices that yield biased (unfair) prices & incentives.

II) We propose a stochastic clearing formulation in which deviations between day-ahead & real-time variables are properly penalized to achieve fair pricing & incentives.

III) Comparisons of deterministic & stochastic settings based on social surplus alone are insufficient to fully appreciate the benefits of stochastic optimization. We present new metrics.

Proposal : Stochastic Clearing

$$
\begin{aligned}
\min_{d_j, D_j(\cdot), g_i, G_i(\cdot), f_\ell, F_\ell(\cdot)} \varphi^{sto} :=\ & \mathbb{E}\left[\sum_{i\in\mathcal{G}} \alpha_i^g G_i(\omega) + \Delta\alpha_i^g |G_i(\omega) - g_i|\right] \\
& - \mathbb{E}\left[\sum_{j\in\mathcal{D}} \alpha_j^d D_j(\omega) - \Delta\alpha_j^d |D_j(\omega) - d_j|\right] \\
& + \mathbb{E}\left[\sum_{\ell\in\mathcal{L}} \Delta\alpha_\ell^f |F_\ell(\omega) - f_\ell|\right] \\
\text{s.t.} \sum_{\ell\in\mathcal{L}_n^{rec}} f_\ell - \sum_{\ell\in\mathcal{L}_n^{snd}} f_\ell + \sum_{i\in\mathcal{G}_n} g_i - \sum_{i\in\mathcal{D}_n} d_i = 0,\ (\pi_n) \qquad & n \in \mathcal{N} \\
\sum_{\ell\in\mathcal{L}_n^{rec}} (F_\ell(\omega) - f_\ell) - \sum_{\ell\in\mathcal{L}_n^{snd}} (F_\ell(\omega) - f_\ell) + \sum_{i\in\mathcal{G}_n} (G_i(\omega) - g_i) & \\
- \sum_{j\in\mathcal{D}_n} (D_j(\omega) - d_j) = 0,\ (p(\omega)\Pi_n(\omega)) \qquad & \omega \in \Omega, n \in \mathcal{N} \\
-\bar{F}_\ell(\omega) \le F_\ell(\omega) \le \bar{F}_\ell(\omega), \qquad & \omega \in \Omega, \ell \in \mathcal{L} \\
0 \le G_i(\omega) \le \bar{G}_i(\omega), \qquad & \omega \in \Omega, i \in \mathcal{G} \\
0 \le D_i(\omega) \le \bar{D}_j(\omega), \qquad & \omega \in \Omega, j \in \mathcal{D}
\end{aligned}
$$

Features:

-Consistency Between Day-Ahead & Real-Time Market

-Differences Between Day-Ahead & Real-Time Variables (Including Flows) Are Penalized

-Does Not Impose Constraints on Day-Ahead Variables (All Information Contained in Scenarios)

Expected Social Surplus *Pritchard, Zakeri & Philpott, 2011*

Definition: The expected social suplus function is defined as:

$$\varphi := \mathbb{E}\left[\sum_{i\in\mathcal{G}} C_i^g(\omega) + \sum_{j\in\mathcal{D}} C_j^d(\omega)\right]$$

$$\begin{aligned} \text{where } C_i^g(\omega) &= +\alpha_i^g g_i + \alpha_i^{g,+}(G_i(\omega)-g_i)_+ - \alpha_i^{g,-}(G_i(\omega)-g_i)_- \\ C_j^d(\omega) &= -\alpha_j^d d_i + \alpha_j^{d,+}(D_j(\omega)-d_j)_- - \alpha_j^{d,-}(D_j(\omega)-d_j)_+, \end{aligned}$$

and $(X-x)_+ = \max(X-x,0)$, $(X-x)_- = \max(x-X,0)$.

Property: Assume bid prices satisfy $\alpha_i^{g,+} - \alpha_i^g = \alpha_i^g - \alpha_i^{g,-} = \Delta\alpha_i^g$ and $\alpha_j^{d,+} - \alpha_j^d = \alpha_j^d - \alpha_j^{d,-} = \Delta\alpha_j^d$. We refer to $\Delta\alpha_i^g$ and $\Delta\alpha_j^d$ as the incremental bid prices. The costs functions become,

$$\begin{aligned} C_i^g(\omega) &:= +\alpha_i^g G_i(\omega) + \Delta\alpha_i^g|G_i(\omega)-g_i|, \quad i\in\mathcal{G},\omega\in\Omega \\ C_j^d(\omega) &:= -\alpha_j^d D_j(\omega) + \Delta\alpha_j^d|D_j(\omega)-d_j|, \quad j\in\mathcal{D},\omega\in\Omega. \end{aligned}$$

Proof:

$$\begin{aligned} C_i^g(\omega) &= \alpha_i^g g_i + \alpha_i^{g,+}(G_i(\omega)-g_i)_+ - \alpha_i^{g,-}(G_i(\omega)-g_i)_- \\ &= \alpha_i^g g_i + (\alpha_i^g + \Delta\alpha_i^g)(G_i(\omega)-g_i)_+ - (\alpha_i^g - \Delta\alpha_i^g)(G_i(\omega)-g_i)_- \\ &= \alpha_i^g g_i + \alpha_i^g(G_i(\omega)-g_i)_+ - \alpha_i^g(G_i(\omega)-g_i)_- + \Delta\alpha_i^g(G_i(\omega)-g_i)_+ + \Delta\alpha_i^g(G_i(\omega)-g_i)_- \\ &= \alpha_i^g g_i + \alpha_i^g(G_i(\omega)-g_i) + \Delta\alpha_i^g|G_i(\omega)-g_i| \\ &= \alpha_i^g G_i(\omega) + \Delta\alpha_i^g|G_i(\omega)-g_i|. \end{aligned}$$

Price Distortions, Premia, & Fairness

Definition: The price distortions or price premia are defined as the difference between day-ahead and expected real-time prices

$$\mathcal{M}_n^\pi := \pi_n - \mathbb{E}\left[\Pi_n(\omega)\right], \quad n \in \mathcal{N}.$$

We say that prices are fair if the premia are zero. We define the average and maximum price premia in the network as,

$$\mathcal{M}^\pi := \frac{1}{|\mathcal{N}|} \sum_{n \in \mathcal{N}} |\mathcal{M}_n^\pi|$$

$$\mathcal{M}_\infty^\pi := \max_{n \in \mathcal{N}} |\mathcal{M}_n^\pi|.$$

Expected Payments

Definition: The expected payments to suppliers and from consumers are:

$$\begin{aligned} \mathbb{E}\left[P_i^g(\omega)\right] &:= g_i(\pi_{n(i)} - \mathbb{E}[\Pi_{n(i)}(\omega)]) + \mathbb{E}\left[G_i(\omega)\Pi_{n(i)}(\omega)\right], \qquad i \in \mathcal{G} \\ \mathbb{E}\left[P_j^d(\omega)\right] &:= d_j(\mathbb{E}[\Pi_{n(j)}(\omega)] - \pi_{n(j)}) - \mathbb{E}\left[D_j(\omega)\Pi_{n(j)}(\omega)\right], \quad j \in \mathcal{D}. \end{aligned}$$

where,

$$\begin{aligned} P_i^g(\omega) &:= g_i\pi_{n(i)} + (G_i(\omega) - g_i)\Pi_{n(i)}(\omega) \\ &= g_i(\pi_{n(i)} - \Pi_{n(i)}(\omega)) + G_i(\omega)\Pi_{n(i)}(\omega), \quad i \in \mathcal{G}, \omega \in \Omega \\ P_j^d(\omega) &:= -d_j\pi_{n(i)} - (D_j(\omega) - d_j)\Pi_{n(j)}(\omega) \\ &= d_j(\Pi_{n(i)}(\omega) - \pi_{n(i)}) - D_j(\omega)\Pi_{n(j)}(\omega), \quad j \in \mathcal{D}, \omega \in \Omega. \end{aligned}$$

Property: If the premia are zero, the expected payments satisfy

$$\begin{aligned} \mathbb{E}\left[P_i^g(\omega)\right] &= +\mathbb{E}\left[G_i(\omega)\Pi_{n(i)}(\omega)\right], \quad i \in \mathcal{G} \\ \mathbb{E}\left[P_j^d(\omega)\right] &= -\mathbb{E}\left[D_j(\omega)\Pi_{n(j)}(\omega)\right], \quad j \in \mathcal{D}. \end{aligned}$$

Relevance of Fair Prices

$$\mathbb{E}\left[P_i^g(\omega)\right] := +g_i(\pi_{n(i)} - \mathbb{E}[\Pi_{n(i)}(\omega)]) + \mathbb{E}\left[G_i(\omega)\Pi_{n(i)}(\omega)\right], \qquad i \in \mathcal{G}$$
$$\mathbb{E}\left[P_j^d(\omega)\right] := -d_j(\pi_{n(j)} - \mathbb{E}[\Pi_{n(j)}(\omega)]) - \mathbb{E}\left[D_j(\omega)\Pi_{n(j)}(\omega)\right], \quad j \in \mathcal{D}.$$

- **Suppliers Benefit More from Positive Premia than Consumers**
- **Fairness Implies that the Market, In Expectation, Behaves as a Pure Real-Time Market**
  - **Prevents Day-Ahead Market From Interfering with Real-Time Market** ***Kaye, 1990***

$$\mathbb{E}\left[P_i^g(\omega)\right] = +\mathbb{E}\left[G_i(\omega)\Pi_{n(i)}(\omega)\right], \quad i \in \mathcal{G}$$
$$\mathbb{E}\left[P_j^d(\omega)\right] = -\mathbb{E}\left[D_j(\omega)\Pi_{n(j)}(\omega)\right], \quad j \in \mathcal{D}.$$

- **How To Enforce Fairness Throughout the System?**

Uplift Payments

Definition: We say that suppliers and consumers are whole in expectation if,

$$\mathbb{E}\left[P_i^g(\omega)\right] \geq \mathbb{E}\left[C_i^g(\omega)\right], \quad i \in \mathcal{G}$$
$$\mathbb{E}\left[P_j^d(\omega)\right] \leq \mathbb{E}\left[C_j^d(\omega)\right], \quad j \in \mathcal{D}.$$

We define the expected uplift payments as,

$$\mathcal{M}_i^U := -\min\left\{\mathbb{E}[P_i^g(\omega)] - \mathbb{E}[C_i^g(\omega)], 0\right\}, \quad i \in \mathcal{G}$$
$$\mathcal{M}_j^U := -\min\left\{\mathbb{E}[C_j^d(\omega)] - \mathbb{E}[P_j^d(\omega)], 0\right\}, \quad j \in \mathcal{D}.$$

The total uplift is $\mathcal{M}^U := \sum_{i\in\mathcal{G}} \mathcal{M}_i^U + \sum_{j\in\mathcal{D}} \mathcal{M}_j^U$.

ISO Revenue Adequacy

Definition: The expected ISO revenue is defined as,

$$\begin{aligned}\mathcal{M}^{ISO} &:= \mathbb{E}\left[\sum_{i\in\mathcal{G}} P_i^g(\omega) - \sum_{j\in\mathcal{D}} P_j^d(\omega)\right] \\ &= \sum_{i\in\mathcal{G}} \mathbb{E}[P_i^g(\omega)] - \sum_{j\in\mathcal{D}} \mathbb{E}[P_j^d(\omega)].\end{aligned}$$

The ISO is said to be revenue adequate in expectation if $\mathcal{M}^{ISO} \leq 0$.

Properties : Pricing (Single Node)

$$\min_{d_j, g_i, G_i(\cdot), D(\cdot)} \mathbb{E}\left[\sum_{i\in\mathcal{G}} \alpha_i^g G_i(\omega) + \Delta\alpha_i^g |G_i(\omega) - g_i|\right] - \mathbb{E}\left[\sum_{j\in\mathcal{D}} \alpha_j^d D_j(\omega) - \alpha_j^d |D_j(\omega) - d_j|\right]$$

$$\text{s.t.} \sum_{i\in\mathcal{G}} g_i = \sum_{j\in\mathcal{D}} d_j \quad (\pi)$$

$$\sum_{i\in\mathcal{G}} (G_i(\omega) - g_i) = \sum_{j\in\mathcal{D}} (D_j(\omega) - d_j) \quad \omega \in \Omega \quad (p(\omega)\Pi(\omega))$$

$$0 \le G_i(\omega) \le \bar{G}_i(\omega), \quad i \in \mathcal{G}, \omega \in \Omega$$

$$0 \le D_j(\omega) \le \bar{D}(\omega), \quad j \in \mathcal{D}, \omega \in \Omega.$$

Theorem S-a: Assume that the incremental bid prices satisfy $\Delta\alpha_j^d > 0,\ j \in \mathcal{D}$ and $\Delta\alpha_i^g > 0,\ i \in \mathcal{G}$. The price distortion $\mathcal{M}^\pi = \pi - \mathbb{E}[\Pi(\omega)]$ is bounded by the incremental bid prices as

$$|\mathcal{M}^\pi| \le \Delta\alpha,$$

where,

$$\Delta\alpha = \min\left\{\min_{i\in\mathcal{G}} \Delta\alpha_i^g, \min_{j\in\mathcal{D}} \Delta\alpha_j^d\right\}.$$

Properties : Pricing (Single Node)

Proof: Consider the partial Lagrange function:

$$
\begin{aligned}
&\mathcal{L}(d_j, D_j(\cdot), g_i, G_i(\cdot), \pi, \Pi(\cdot)) = \\
&\mathbb{E}\left[\sum_{i\in\mathcal{G}} \alpha_i^g G_i(\omega) + \Delta\alpha_i^g |G_i(\omega) - g_i|\right] - \mathbb{E}\left[\sum_{j\in\mathcal{D}} \alpha_j^d D_j(\omega) - \alpha_j^d |D_j(\omega) - d_j|\right] \\
&- \pi\left(\sum_{i\in\mathcal{G}} g_i - \sum_{j\in\mathcal{D}} d_j\right) - \mathbb{E}\left[\Pi(\omega)\left(\sum_{i\in\mathcal{G}}(G_i(\omega) - g_i) - \sum_{j\in\mathcal{D}}(D_j(\omega) - d_j)\right)\right].
\end{aligned}
$$

The stationarity conditions w.r.t. day-ahead quantities are:

$$
\begin{aligned}
\partial_{d_j}\mathcal{L} =& 0 = \Delta\alpha_j^d \mathbb{P}(d_j \geq D_j(\omega)) - \Delta\alpha_j^d \mathbb{P}(d_j \leq D_j(\omega)) + \pi - \mathbb{E}\left[\Pi(\omega)\right] \quad j \in \mathcal{D} \\
\partial_{g_i}\mathcal{L} =& 0 = \Delta\alpha_i^g \mathbb{P}(g_i \geq G_i(\omega)) - \Delta\alpha_i^g \mathbb{P}(g_i \leq G_i(\omega)) - \pi + \mathbb{E}\left[\Pi(\omega)\right] \quad i \in \mathcal{G}.
\end{aligned}
$$

Where $\mathbb{P}(A)$ denotes the probability of event A. This results from

$$
\partial_x |X - x| = \begin{cases} +1 \text{ if } X < x \\ -1 \text{ if } X > x. \end{cases}
$$

and
$$
\begin{aligned}
\partial_x \mathbb{E}[|X(\omega) - x|] &= \mathbb{E}\left[\mathbf{1}_{X(\omega)<x} - \mathbf{1}_{X(\omega)>x}\right] \\
&= \mathbb{E}\left[\mathbf{1}_{X(\omega)<x} + \mathbf{1}_{X(\omega)=x} - \mathbf{1}_{X(\omega)=x} - \mathbf{1}_{X(\omega)>x}\right] \\
&= \mathbb{P}(X(\omega) \leq x) - \mathbb{P}(X(\omega) \geq x).
\end{aligned}
$$

Properties : Pricing (Single Node)

Proof (continuation): Rearranging stationarity conditions we obtain,

$$\mathbb{P}(d_j \geq D_j(\omega)) = \frac{\Delta\alpha_j^d - \pi + \mathbb{E}\left[\Pi(\omega)\right]}{2\Delta\alpha_j^d}$$
$$\mathbb{P}(g_i \geq G_i(\omega)) = \frac{\Delta\alpha_i^g + \pi - \mathbb{E}\left[\Pi(\omega)\right]}{2\Delta\alpha_i^g}$$

Because $\sum_{\omega\in\Omega} p(\omega) = 1$, we have the implicit probability bounds $0 \leq \mathbb{P}(d_j \geq D_j(\omega)) \leq 1$ and $0 \leq \mathbb{P}(g_i \geq G_i(\omega)) \leq 1$. We thus have,

$$0 \leq \frac{\Delta\alpha_j^d - \pi + \mathbb{E}\left[\Pi(\omega)\right]}{2\Delta\alpha_j^d} \leq 1$$
$$0 \leq \frac{\Delta\alpha_i^g + \pi - \mathbb{E}\left[\Pi(\omega)\right]}{2\Delta\alpha_i^g} \leq 1.$$

These are equivalent to,

$$|\pi - \mathbb{E}\left[\Pi(\omega)\right]| \leq \Delta\alpha_j^d, \qquad j \in \mathcal{D}$$
$$|\pi - \mathbb{E}\left[\Pi(\omega)\right]| \leq \Delta\alpha_i^g, \qquad i \in \mathcal{G}.$$

The proof is complete. $\square$

Properties : Convergence of Day-Ahead Quantities (Single Node)

Theorem S-b: If the price distortion $\mathcal{M}^{\pi}$ is zero at the solution, the day-ahead quantities converge to the median of the real-time quantities,

$$d_j = \mathbb{M}[D_j(\omega)], \; j \in \mathcal{D}$$
$$g_i = \mathbb{M}[G_i(\omega)], \; i \in \mathcal{G}.$$

Proof: We go back to,

$$\mathbb{P}(d_j \geq D_j(\omega)) = \frac{\Delta\alpha_j^d - \pi + \mathbb{E}\left[\Pi(\omega)\right]}{2\Delta\alpha_j^d}$$
$$\mathbb{P}(g_i \geq G_i(\omega)) = \frac{\Delta\alpha_i^g + \pi - \mathbb{E}\left[\Pi(\omega)\right]}{2\Delta\alpha_i^g}.$$

If the price distortion is zero then,

$$\mathbb{P}(d_j \geq D_j(\omega)) = \frac{1}{2}$$
$$\mathbb{P}(g_i \geq G_i(\omega)) = \frac{1}{2}.$$

This implies that $\mathbb{P}(d_j \geq D_j(\omega)) = \mathbb{P}(d_j \leq D_j(\omega))$ and $d_j = \mathbb{M}[D_j(\omega)]$. The same holds for suppliers. $\square$

Properties : Bounds on Day-Ahead Quantities (Single Node)

Theorem S-c: The day-ahead quantities are bounded by the real-time quantities as,

$$\min_{\omega \in \Omega} D_j(\omega) \leq d_j \leq \max_{\omega \in \Omega} D_j(\omega), \; j \in \mathcal{D}$$
$$\min_{\omega \in \Omega} G_i(\omega) \leq g_i \leq \max_{\omega \in \Omega} G_i(\omega), \; i \in \mathcal{G}.$$

Proof: From the implicit probability bounds we have that,

$$-\Delta\alpha_j^d \leq \pi - \mathbb{E}\left[\Pi(\omega)\right] \leq \Delta\alpha_j^d$$
$$-\Delta\alpha_i^g \leq \pi - \mathbb{E}\left[\Pi(\omega)\right] \leq \Delta\alpha_i^g.$$

Consider the case in which the distortion hits the lower bound $\pi - \mathbb{E}\left[\Pi(\omega)\right] = -\Delta\alpha_j^d$. From,

$$\mathbb{P}(d_j \geq D_j(\omega)) = \frac{\Delta\alpha_j^d - \pi + \mathbb{E}\left[\Pi(\omega)\right]}{2\Delta\alpha_j^d}$$

we have that $\mathbb{P}(d_j \geq D_j(\omega)) = 1$. This implies that $d_j \geq D_j(\omega), \; \forall \omega \in \Omega$ and $d_j \geq \min_\omega D_j(\omega) \geq 0$. If $\pi - \mathbb{E}\left[\Pi(\omega)\right] = +\Delta\alpha_j^d$ we have $\mathbb{P}(d_j \leq D_j(\omega)) = 1$. This implies that $d_j \leq D_j(\omega), \; \forall \omega \in \Omega$ and $d_j \leq \max_\omega D_j(\omega)$. Same procedure is used for suppliers. $\square$

Properties : Zero Uplift (Single Node)

Theorem S-d: Any minimizer $d_j^*, D_j(\cdot)^*, g_i^*, G_i(\cdot)^*, \pi^*, \Pi^*(\cdot)$ yields zero uplift payments in expectation,

$$\mathcal{M}_i^U = 0, \quad i \in \mathcal{G}$$
$$\mathcal{M}_j^U = 0, \quad j \in \mathcal{D}.$$

Proof: Because the problem is convex, we know that the prices $\pi^*, \Pi^*(\cdot)$ satisfy:

$$(d_j^*, D_j(\cdot)^*, g_i^*, G_i^*(\cdot)) = \underset{d_j, D_j(\cdot), g_i, G_i(\cdot)}{\text{argmin}} \; \mathcal{L}(d_j, D_j(\cdot), g_i, G_i(\cdot), \pi^*, \Pi^*(\cdot)) \text{ s.t. Bounds.}$$

At $\pi^*, \Pi^*(\cdot)$, the partial Lagrange function is separable and can be written as,

$$\mathcal{L}(d_j, D_j(\cdot), g_i, G_i(\cdot), \pi^*, \Pi^*(\cdot)) = \sum_{i \in \mathcal{G}} \mathcal{L}_i^g(g_i, G_i(\cdot), \pi^*, \Pi^*(\cdot)) - \sum_{j \in \mathcal{D}} \mathcal{L}_j^d(d_j, D_j(\cdot), \pi^*, \Pi^*(\cdot)),$$

where,

$$\mathcal{L}_i^g(g_i, G_i(\cdot), \pi^*, \Pi^*(\cdot)) := \mathbb{E}[C_i^g(\omega)] - \mathbb{E}[P_i^g(\omega)], \; i \in \mathcal{G}$$
$$\mathcal{L}_j^d(d_j, D_j(\cdot), \pi^*, \Pi^*(\cdot)) := \mathbb{E}[P_j^d(\omega)] - \mathbb{E}[C_j^d(\omega)], \; j \in \mathcal{D}.$$

Since zero is a feasible solution, optimality yields the result. $\square$

Properties : Network System

$$
\begin{aligned}
&\mathcal{L}(d_j, D_j(\cdot), g_i, G_i(\cdot), f_\ell, F_\ell(\cdot), \pi_n, \Pi_n(\cdot)) = \\
&\quad \mathbb{E}\left[\sum_{i\in\mathcal{G}} \alpha_i^g G_i(\omega) + \Delta\alpha_i^g |G_i(\omega) - g_i|\right] - \mathbb{E}\left[\sum_{j\in\mathcal{D}} \alpha_j^d D_j(\omega) - \alpha_j^d |D_j(\omega) - d_j|\right] + \sum_{\ell\in\mathcal{L}} \Delta\alpha_\ell^f \mathbb{E}\left[|F_\ell(\omega) - f_\ell|\right] \\
&- \sum_{n\in\mathcal{N}} \pi_n \left(\sum_{\ell\in\mathcal{L}_n^{rec}} f_\ell - \sum_{\ell\in\mathcal{L}_n^{snd}} f_\ell + \sum_{i\in\mathcal{G}_n} g_i - \sum_{i\in\mathcal{D}_n} d_i\right) \\
&- \mathbb{E}\left[\sum_{n\in\mathcal{N}} \Pi_n(\omega) \left(\sum_{\ell\in\mathcal{L}_n^{rec}} (F_\ell(\omega) - f_\ell) - \sum_{\ell\in\mathcal{L}_n^{snd}} (F_\ell(\omega) - f_\ell) + \sum_{i\in\mathcal{G}_n} (G_i(\omega) - g_i) - \sum_{j\in\mathcal{D}_n} (D_j(\omega) - d_j)\right)\right]
\end{aligned}
$$

Theorem N-a: Consider the stochastic network clearing model and assume that the incremental bid costs satisfy $\Delta\alpha_j^d > 0,\ j \in \mathcal{D}$, $\Delta\alpha_i^g > 0,\ i \in \mathcal{G}$, and $\Delta\alpha_\ell^f > 0,\ \ell \in \mathcal{L}$. The price distortions $\mathcal{M}_n^\pi$ are bounded as,

$$
\begin{aligned}
|\mathcal{M}_n^\pi| &\leq \Delta\alpha_n,\ n \in \bar{\mathcal{N}} \\
|\mathcal{M}_{snd(\ell)}^\pi - \mathcal{M}_{rec(\ell)}^\pi| &\leq \Delta\alpha_\ell^f,\ \ell \in \mathcal{L}
\end{aligned}
$$

where,

$$
\Delta\alpha_n = \min\left\{\min_{i\in\mathcal{G}_n} \Delta\alpha_i^g, \min_{j\in\mathcal{D}_n} \Delta\alpha_j^d\right\},\ n \in \bar{\mathcal{N}}.
$$

Properties : Network System

Theorem N-b: If the price distortions $\mathcal{M}_n^\pi,\ n \in \mathcal{N}$ are zero, the day-ahead quantities and flows converge to the medians of real-time counterparts,

$$
\begin{aligned}
d_j &= \mathbb{M}[D_j(\omega)],\ j \in \mathcal{D} \\
g_i &= \mathbb{M}[G_i(\omega)],\ i \in \mathcal{G} \\
f_\ell &= \mathbb{M}[F_\ell(\omega)],\ \ell \in \mathcal{L}.
\end{aligned}
$$

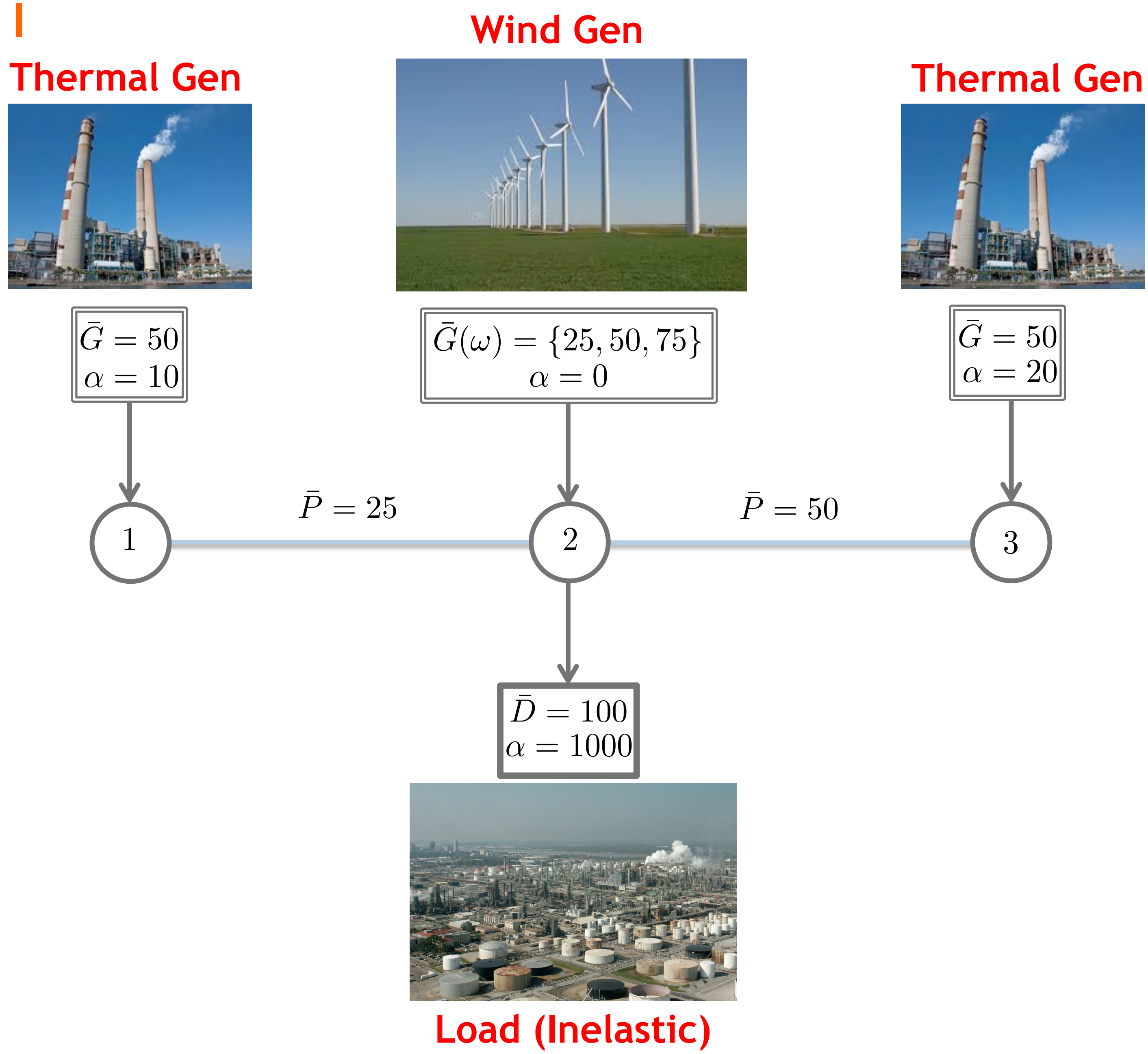
Theorem N-c: The day-ahead quantities and flows are bounded by their real-time counterparts as,

$$
\begin{aligned}
\min_{\omega\in\Omega} D_j(\omega) \le d_j \le \max_{\omega\in\Omega} D_j(\omega),\ j \in \mathcal{D} \\
\min_{\omega\in\Omega} G_i(\omega) \le g_i \le \max_{\omega\in\Omega} G_i(\omega),\ i \in \mathcal{G} \\
\min_{\omega\in\Omega} F_\ell(\omega) \le f_\ell \le \max_{\omega\in\Omega} F_\ell(\omega),\ \ell \in \mathcal{L}.
\end{aligned}
$$

Theorem N-d: Any minimizer $d_j^*, D_j(\cdot)^*, g_i^*, G_i(\cdot)^*, f_\ell^*, F_\ell^*, \pi_n^*, \Pi_n^*(\cdot)$ yields zero uplift payments and revenue adequacy in expectation,

$$
\begin{aligned}
\mathcal{M}^{ISO} &\le 0 \\
\mathcal{M}_i^U &= 0, \quad i \in \mathcal{G} \\
\mathcal{M}_j^U &= 0, \quad i \in \mathcal{D}.
\end{aligned}
$$

Example I



Wind Gen
Thermal Gen
Thermal Gen
$\bar{G} = 50$
$\alpha = 10$
$\bar{G}(\omega) = \{25, 50, 75\}$
$\alpha = 0$
$\bar{G} = 50$
$\alpha = 20$
$\bar{P} = 25$
$\bar{P} = 50$
1
2
3
$\bar{D} = 100$
$\alpha = 1000$
Load (Inelastic)

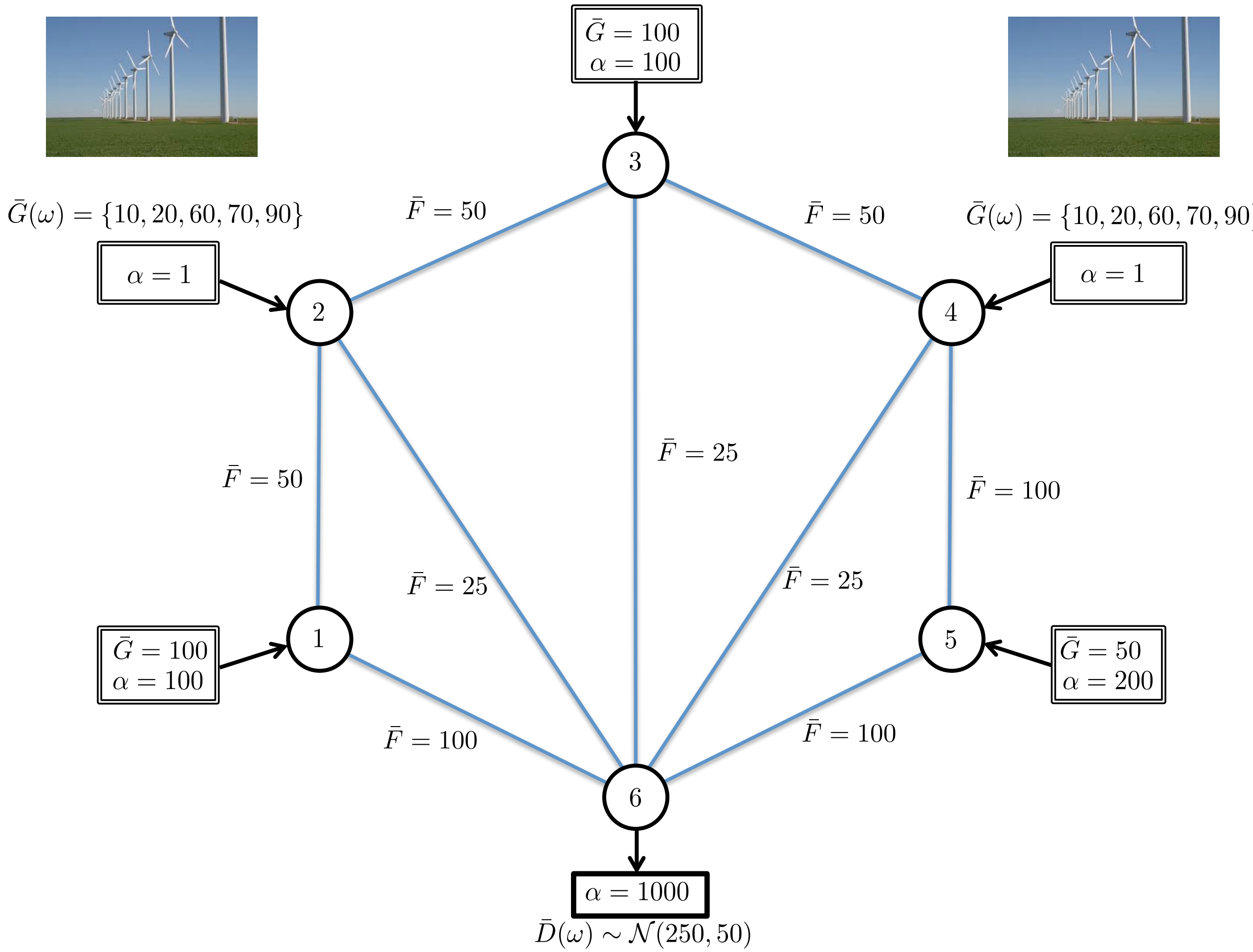
Example I

	Day-Ahead Quantities	Day-Ahead Prices	RT Quantities	RT Prices	Expected RT Prices	Expected Welfare
	g_i	π_n	$G_i(\omega)$	$\Pi_n(\omega)$	$\mathbb{E}[\Pi_n(\omega)]$	φ^{gen}
Deterministic	{25,50,25}	{10,20,20}	{25,25,50} {25,50,25} {25,75,0}	{15,802,412} {15,22,22} {15,18,18}	{15,280,150}	835
Stochastic	{25,50,25}	{10,276,148}	{25,25,50} {25,50,25} {25,75,0}	{10,790,406} {10,20,20} {10,18,18}	{10,276,148}	835
Stochastic-WS	{25,25,50} {25,50,25} {25,75,0}	{10,821,420} {10,20,20} {10,20,20}	{25,25,50} {25,50,25} {25,75,0}	{10,803,412} {10,20,20} {10,20,20}	{10,281,150}	800

	Payments	Costs	ISO Revenue
	$\mathbb{E}[P_i^g(\omega)]$	$\mathbb{E}[C_i^g(\omega)]$	$\mathcal{M}^{ISO}$
Deterministic	{250,-5553,3799}	{250,5,533}	-3504
Stochastic	{250,7316,6886}	{250,5,533}	-12955
Stochastic-WS	{250,7555,7055}	{250,5,500}	-13360

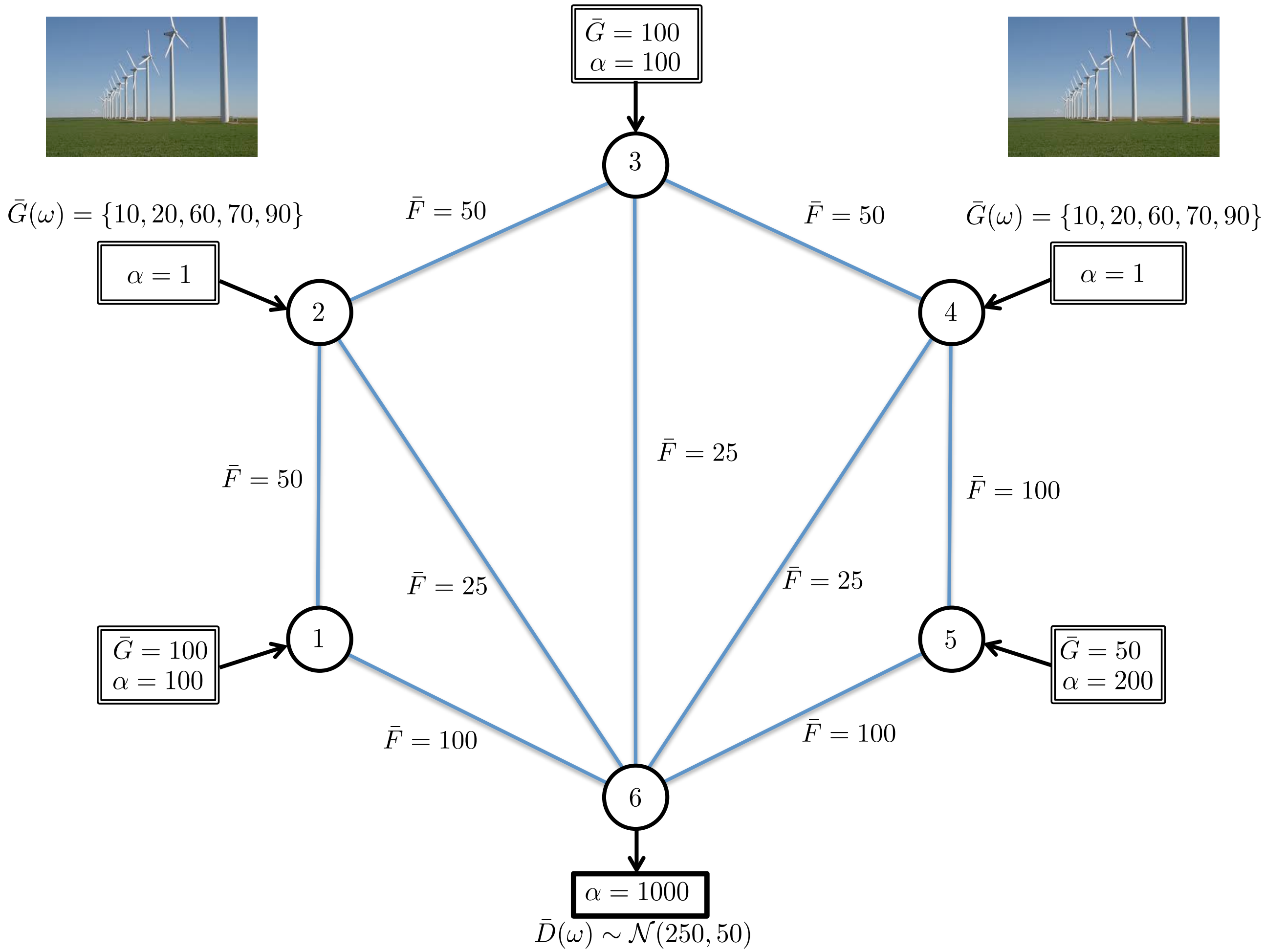
Bid Price $\Delta\alpha^d$	Max Distortion $\mathcal{M}_\infty^\pi$
1.0	0.43
0.1	0.058
0.01	0.006
0.001	0.0006

System II



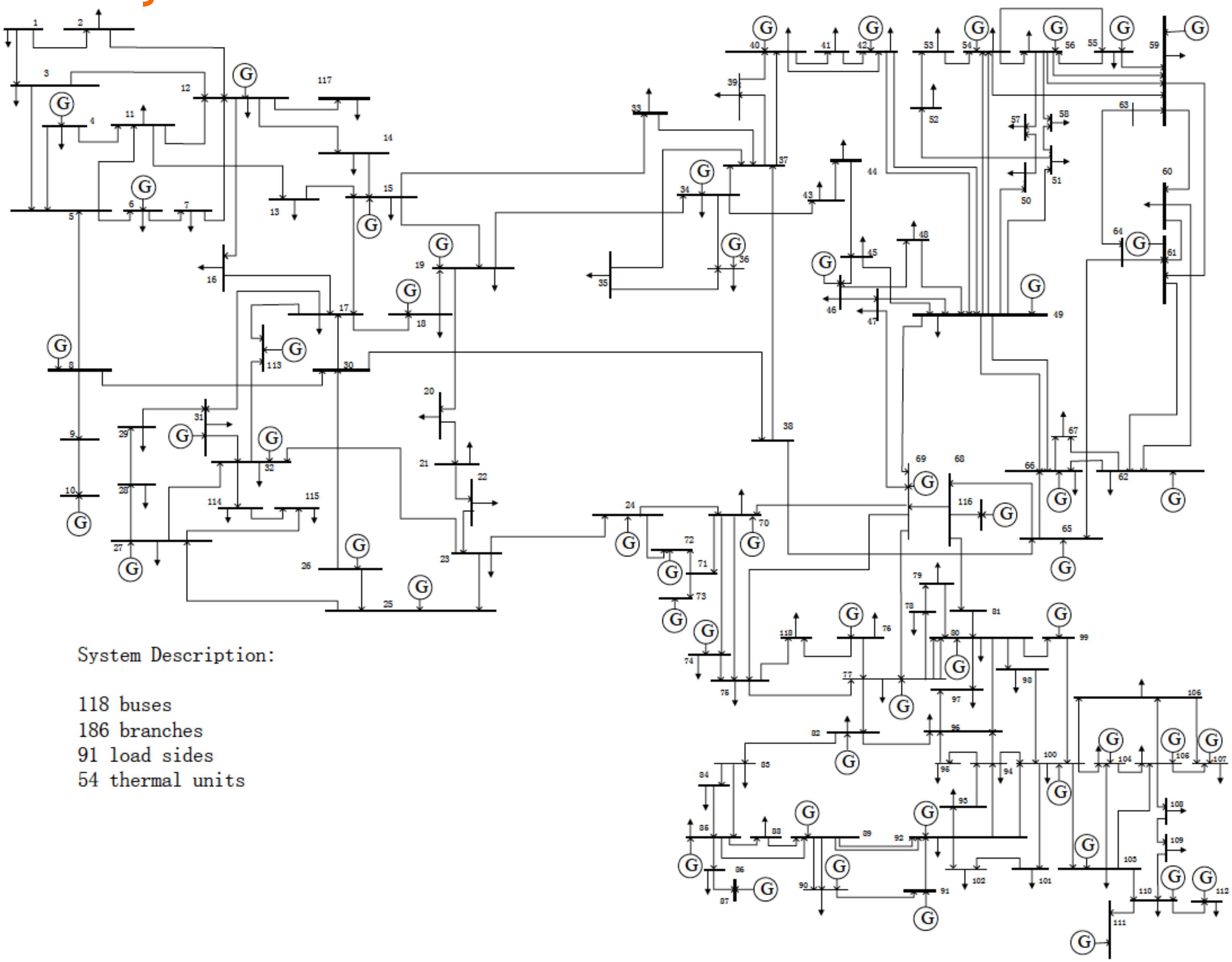
	Social Surplus φ	Per Node Premia $\mathcal{M}_n^\pi$	Day-Ahead Prices π_n
Deterministic	-217529	{9,9,9,-205,-208,-273}	{100,100,100,100,100,100}
Stochastic	-217628	{0.001,0.001,0.001,0,0,0}	{96,96,96,307,310,374}
Stochastic-WS	-218266	-	-

System II (Transmission Contingencies)



	Payments		ISO Revenue
	$\mathbb{E}[P_i^g(\omega)]$	$\mathbb{E}[C_i^g(\omega)]$	$\mathcal{M}^{ISO}$
Deterministic	{5803,4723,5100,-919,9627}	{5231,50,3876,47,3387}	40570
Stochastic	{5107,3955,3683,7371,9623}	{5107,51,3683,46,3383}	-118103
Stochastic-WS	{4951,3888,3422,7170,9479}	{4951,47,3422,45,3079}	-118283

IEEE118 System



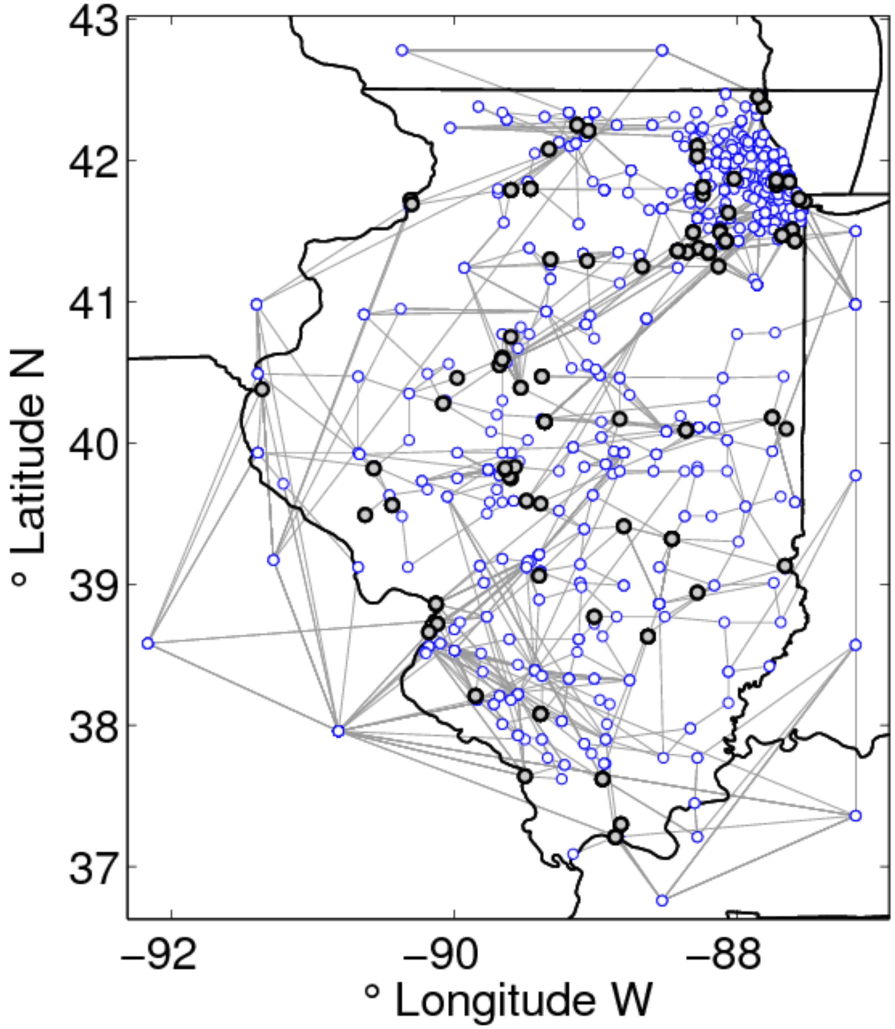
System Description:

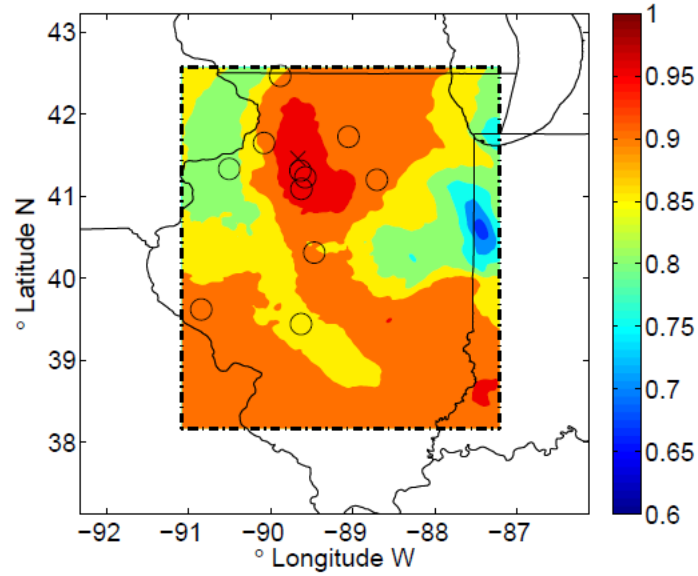
118 buses
186 branches
91 load sides
54 thermal units

IEEE118 System

	Total Uplift			Max Premia	
	$\mathcal{M}^U$	φ^{gen}	$\mathcal{M}^\pi$	$\mathcal{M}^\pi_\infty$	$\mathcal{M}^{ISO}$
Deterministic	-13,797	344,142	16	280	-833,656
Stochastic	0	343,959	0.0005	0.0017	-818,250
Stochastic-WS	0	343,578	0	0	-818,583

Illinois System





Dimensions

1900 Buses

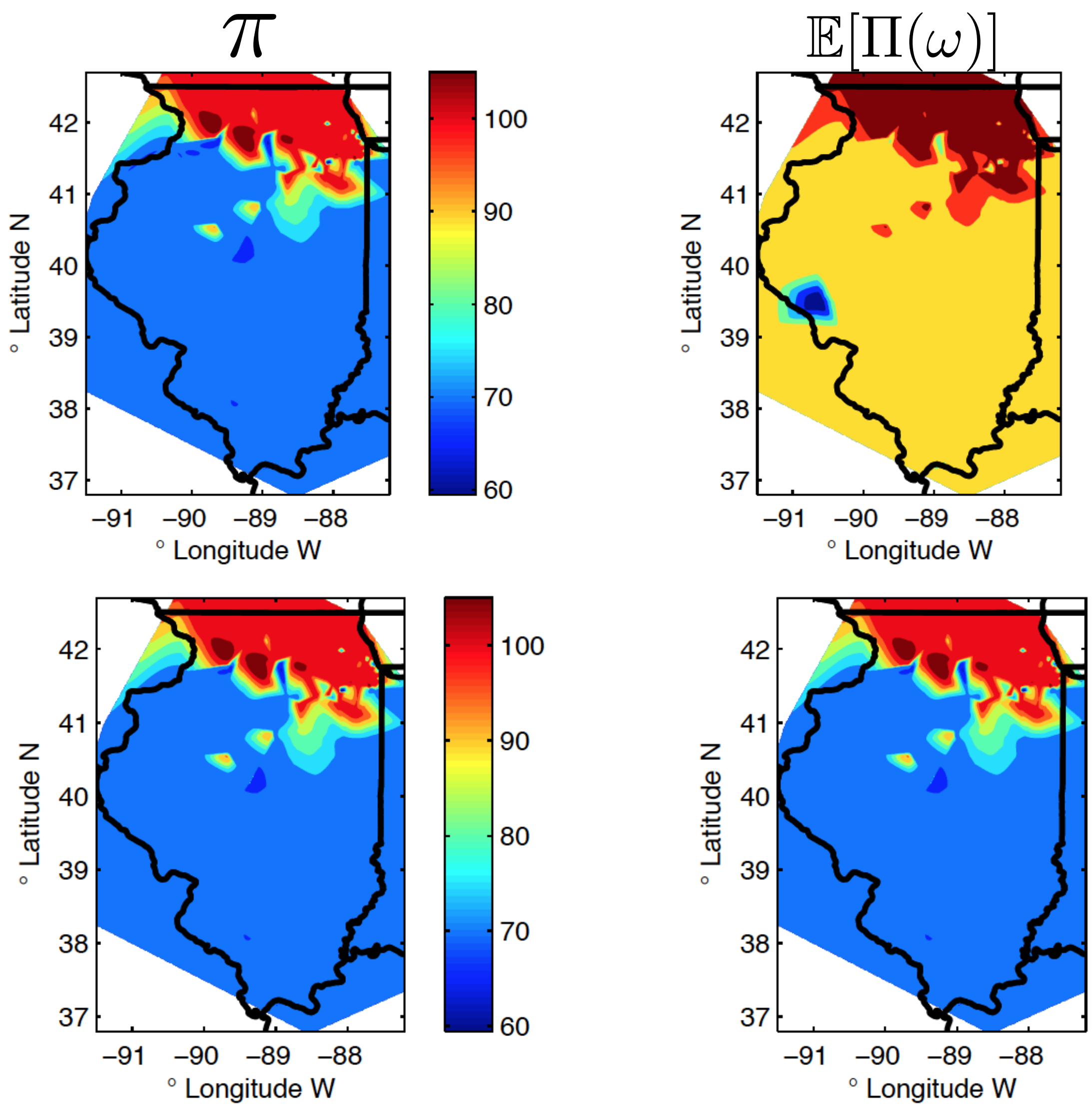
261 Generators

300 Demands

Illinois System

Deterministic

Stochastic



The Message(s)

- **Deterministic clearing introduces strong distortions between day-ahead & expected real-time prices that yield biased (unfair) prices & incentives.**

- **We proposed a stochastic clearing formulation in which deviations between day-ahead & real-time variables are properly penalized to achieve fair pricing & incentives.**

- **We showed that the stochastic clearing formulation has several good properties, such as revenue adequacy in expectation and bounded price distortions.**

- **We demonstrated through numerical simulations that the price distortions are much reduced compared to the deterministic case.**

Open Questions

How to Manage Uncertainty in a Decentralized Market?

How to Solve Stochastic Problems with Large Coupling?

How to Extend Analysis to More Complicated Settings (Multistage)?